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Observations of Ionic Sound Waves in Plasmas: Their Properties and Applications

I. ALEXEFF AND R. V. NEIDIGH
Oak Ridge National Laboratory, Oak Ridge, Tennessee*
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This paper discusses extensive experimental studies of the ionic sound waves predicted by Tonks and Langmuir. Self-excited oscillations have been observed both in magnetically supported plasma columns and in spherical discharge tubes. The fundamental frequency and the overtones of each system agree with those predicted by the ionic-sound-wave formula in terms of the electron temperature, the ion mass, and the dimensions of the system. A simple equation is used to predict when damping by the neutral gas prevents a given mode of plasma oscillation from being observed. The thermal electron pressure, required for the existence of an ionic sound wave, has been measured mechanically. Resonance effects have been observed between the ionic-sound and the ion cyclotron frequency in the magnetically supported plasma column. Practical applications of ionic sound waves are as follows: The electron temperature of a plasma was found by measuring the frequency of a standing ion-wave mode. The neutral gas density was estimated by observing the gas damping of various standing ion-wave modes.

I. INTRODUCTION

THIS paper describes an extensive experimental study of ionic sound waves in plasmas. An ionic sound wave in a plasma is similar to an ordinary sound wave in a gas. Like ordinary sound, an ionic sound wave is a longitudinal compression wave with the momentum due to the mass density and the restoring force due to thermal pressure.

However, ionic sound waves differ from ordinary sound waves because they occur in plasmas. Unlike a gas, a plasma is composed of two quite different components, electrons and ions. Under the conditions of ionic-sound-wave motion, the electron and the ion fluids are constrained to move as one by electrostatic forces. The ion fluid possesses more mass density and supplies most of the momentum of the wave. In this work, the electron fluid is the hotter, and supplies most of the restoring force of the wave. Thus the velocity of the ionic sound waves described here depends primarily on the mass of the ions and the temperature of the electrons.

In this paper, ionic sound waves are described in both a magnetically supported plasma column formed by a reflex discharge and in the plasma of a spherical discharge tube. In both cases, the electron temperature was between 1 and 50 eV, and the ion temperature was much lower. The ionic sound waves were generated spontaneously by a mechanism that is as yet uninvestigated. Resonant modes of oscillation which restricted the ionic sound to steady oscillations at discrete frequencies were present in both the magnetically supported plasma column and the sphere.

The observation and the theoretical prediction of the discrete frequencies are the main subjects of this paper. In addition, there are discussions of some observations of plasma phenomena which are closely connected with the ionic sound waves.

The paper has the following pattern: First, the theoretical properties of ionic sound waves are discussed. A simple equation is derived here, which predicts when damping by the residual gas prevents a given mode of plasma oscillation from being observed. Next, the observations of other workers are mentioned. Then, the authors' experimental work is discussed. Observations of self-excited ionic sound oscillations were made in a magnetically supported plasma column. In the absence of an external magnetic field, ionic sound oscillations were observed in spherical discharge tubes. The effect of gas damping is computed for both the plasma column and the discharge tubes. For the discharge tubes the calculations predict serious gas-damping effects, and these effects are apparently observed.

The thermal pressure of the plasma electrons (the restoring force producing the ionic sound waves) was demonstrated by a mechanical-force measurement. Resonance effects between the ionic-sound-wave frequency and the ion cyclotron frequency were noted in the magnetically supported plasma column. The paper terminates with a summary and conclusions.

Practical applications of ionic sound waves are discussed in the following sections: In Sec. IV E, the background gas pressure in a plasma was estimated by observing the gas damping of various standing ion-wave modes. In Sec. VI, the electron temperature in a plasma was computed from the observed frequency of the fundamental mode of ion-wave oscillation.

Parts of this study of ionic sound waves have appeared in short, locally distributed papers. However, in this paper the material is presented in detail and as an integrated whole.

II. THEORETICAL PROPERTIES OF IONIC SOUND WAVES

A. Derivation of Ionic-Sound-Wave Formula

The basic equation which leads to ionic sound waves was first derived by Tonks and Langmuir,[1] and has since been obtained in various ways by more recent

* Operated by Union Carbide Corporation for the U. S. Atomic Energy Commission.

[1] L. Tonks and I. Langmuir, Phys. Rev. 33, 195 (1929).

authors.[2–4] The basic equation is

$$\nu=\left(\frac{ne^2}{\pi m_i+ne^2m_i\lambda^2/\gamma kT_e}\right)^{1/2}, \quad (1)$$

where ν is the frequency in cycles per second, n is the number of electrons per cubic centimeter. (The plasma is assumed to be electrically neutral and the ions singly charged, so that n is also the number of ions per cubic centimeter.) e is the charge of an electron in esu, m_i is the mass of an ion in grams, λ is the wavelength of the ion wave in centimeters, γ is the adiabatic compression coefficient of the electron gas, k is Boltzmann's constant in ergs per °K, and T_e is the electron temperature in °K. This equation is valid only if the electron temperature is much higher than the ion temperature. If the electron temperature is comparable to the ion temperature, the ionic sound wave can be severely damped.[3,4]

If the wavelength of a wave λ is considerably longer than the Debye cutoff length L_d,[5] then the simple equation for ionic sound waves is obtained.

If

$$\lambda\gg L_d=(kT_e/4\pi ne^2)^{1/2}, \quad (2)$$

then

$$\nu=(1/\lambda)(\gamma kT_e/m_i)^{1/2}. \quad (3)$$

Both the derivation of the above formula and the name "ionic sound wave" appear in the original work of Tonks and Langmuir.[1] In this paper, we will discuss only the simple case of ionic sound waves, those waves given by Eq. (3).

The constant γ in the ionic-sound-wave formula refers to the adiabatic compression coefficient of the electron gas in the plasma. In the paper of Tonks and Langmuir,[1] γ was set equal to 1, which corresponds to isothermal compression. If the electrons suffer scatterings but no energy losses during a cycle of the ionic sound wave, then γ is 5/3. This value of γ corresponds to an ordinary sound wave in a gas. If the plasma is sufficiently rarefied, the electrons suffer no collisions with other particles during one cycle of the ionic sound wave. The electrons are compressed only by electric fields. In this case, discussed by Spitzer,[6] the compression is one dimensional and γ is 3. Values of γ lying between 1, 5/3, and 3 may occur, but for these intermediate values of γ, sound waves are strongly damped.[7]

B. Derivation of the Electron Pressure of a Plasma

A plasma can exhibit a thermal electron pressure, which is quite analogous to the thermal pressure of molecules in an ordinary gas. If the ionic-sound-wave formula, Eq. (3), is valid, then the existence of the thermal electron pressure can be shown as follows: Let the numerator and the denominator inside the square root of Eq. (3) be multiplied by the number of electrons or ions per cubic centimeter, n. Next, multiply both sides of the equation by λ, and finally substitute $v=\nu\lambda$ on the left-hand side, where v is the velocity of the ionic sound wave in centimeters per second. The equation now has the form

$$v=(\gamma nkT_e/nm_i)^{1/2}. \quad (4)$$

Dimensionally, the quantity in parentheses has the form of a pressure term divided by a mass density term. The form of Eq. (4) is now quite analogous to the equation for the velocity v_g of an ordinary sound wave in a gas:

$$v_g=(\gamma p/\sigma)^{1/2}. \quad (5)$$

In this equation, σ is the mass per unit volume and p is the pressure per unit area. In the case of a plasma [Eq. (4)], the term nm_i is mass per unit volume due to the ions σ_i. Therefore, the term in the numerator should represent an actual thermal pressure p_e in dynes per square centimeter exerted by the plasma electrons. The thermal pressure of the plasma electrons therefore should be

$$p_e=nkT_e. \quad (6)$$

C. Discussion of Gas Damping

Ionic sound waves can be strongly damped by un-ionized gas present in the plasma. Gas damping occurs because collisions between the plasma ions and the gas atoms rob the ions of their coherent ionic-sound-wave motion. The effects of gas damping in plasmas are easily discussed quantitatively.

In discussing gas damping, some assumptions are needed to define the system. The following assumptions define a plasma similar to that in an ordinary gaseous discharge tube. Ions and gas atoms are assumed to have the same mass. A second assumption is that the coherent velocity of the ions due to the ionic-sound-wave motion is small compared to the thermal, random velocity of the ions.

Under the above assumptions, the average frequency of collision of an ion with gas atoms is independent of the ionic-sound-wave motion. The rate of collision of an ion with gas atoms is given by the equation[8]

$$\nu_c=\sqrt{2}\sigma n_g(1.13)(2kT/m)^{1/2}. \quad (7)$$

Here, ν_c is the average number of collisions per second made by one ion with gas atoms, σ is the collision cross section in square centimeters between an ion and a gas atom, n_g is the number of gas atoms per cubic centimeter in the plasma, k is Boltzmann's constant in ergs per °K, m is the mass of an ion or an atom in grams, and T is

[2] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), p. 58.
[3] B. D. Fried and R. W. Gould, Phys. Fluids **4**, 139 (1961).
[4] I. B. Bernstein and R. M. Kulsrud, Phys. Fluids **3**, 937 (1960); **4**, 1037 (1961); **5**, 210 (1962).
[5] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), p. 16.
[6] Reference 5, p. 13.
[7] J. W. S. Rayleigh, *The Theory of Sound* (Dover Publications, New York, 1956), Sec. 247.

[8] G. Joos, *Theoretical Physics* (Hafner Publishing Company, New York, 1950), pp. 559 and 585.

the temperature of the ions and the gas atoms in °K. In this equation, the temperature of the ions is assumed to be equal to the temperature of the gas atoms. However, if the ion temperature is much higher than the gas atom temperature, Eq. (7) is modified by removing the factor $\sqrt{2}$ and by using the ion temperature for T.

Every time an ion collides with a gas atom, the ion is assumed to lose all the energy which it could return to the ionic sound wave. This assumption is not strictly correct, for the ion may loose only part of its energy. However, the assumption is not too unrealistic because the fraction of energy lost per collision is large—it is made in order to simplify the calculation.

A large energy loss occurs in a collision between an ion and a gas atom because two kinds of energy-loss effects are present. The first effect is due merely to the ion's losing kinetic energy and slowing down. The second effect is due to the angular deflection of the ion, for only the ion momentum parallel to the ionic-sound-wave motion belongs to the sound wave. Thus, the scattering of an ion elastically through a right angle by a much heavier gas atom would not reduce the ion's kinetic energy appreciably, but would deprive the ionic sound wave of all the wave energy that the ion had possessed. An example of the two kinds of energy loss can be computed for hard-sphere scattering. For a moving sphere incident on a stationary sphere of equal mass, the average fraction of the total kinetic energy lost is 1/2. However, the average fraction of the kinetic energy lost from the motion in the initial direction is 2/3, which is large.

Under the above assumptions, the rate at which an ionic sound wave loses energy is calculated. Half the total energy of a sound wave is stored in the kinetic energy of moving mass, on the average. This fact is easily shown by elementary analysis of sound wave motion. Thus, the average sound wave energy stored in the motion of one ion in a cubic centimeter of oscillating plasma is

$$E_i = E/2n. \tag{8}$$

In this equation, E_i is the ion energy in ergs, E is the energy of the ionic sound wave in ergs per cubic centimeter, and n is the number of ions per cubic centimeter. The rate at which an ion loses energy to the gas atoms is the average energy that the ion possesses times the rate at which it collides with the gas atoms:

$$-dE_i/dt = (E/2n)\nu_c. \tag{9}$$

Here $-dE_i/dt$ is the energy loss rate per ion in ergs per second. The rate at which the plasma volume loses energy is the rate of loss for one ion times the number of ions in the volume:

$$-dE/dt = -n dE_i/dt = \tfrac{1}{2}E\nu_c. \tag{10}$$

Here $-dE/dt$ is the energy loss rate for all the ions in the plasma volume in ergs per cubic centimeter per second.

Next, one evaluates the damping for an ionic sound wave. Consider, for example, a resonant system formed by a standing ionic sound wave in some apparatus. The Q (quality factor) of the system can be defined as 2π times the energy stored per cycle of the undamped wave divided by the energy dissipated per cycle. The value of Q for a standing ionic sound wave, assuming uniform plasma and background gas density, is

$$Q = \frac{2\pi E}{-(dE/dt)1/\nu} = \frac{4\pi\nu}{\nu_c}. \tag{11}$$

Here ν is the frequency of the undamped ionic sound wave in cycles per second, as given by Eq. (3). As Q becomes smaller, the damping of the wave increases until for $Q=1/2$ the system is critically damped. For critical damping, the system does not show oscillatory behavior.

For a given standing ionic-sound-wave mode to appear in a system, the Q of the mode must be greater than 1/2. As Eq. (11) shows, the higher frequency modes have higher values of Q. Thus, in a system containing much un-ionized gas, the lower frequency standing wave modes are damped out. High-frequency modes might be excited, but they have frequencies very close to each other. Thus, standing ionic sound waves in a system containing much gas would probably produce a spectrum which appears to be a structureless continuum.

III. HISTORY OF EXPERIMENTAL IONIC SOUND STUDIES

In this brief review, emphasis is placed on those papers which clearly demonstrate the ionic-sound dispersion relation.

$$\nu = \lambda^{-1}(\gamma k T_e)^{1/2} m_i^{-1/2}.$$

The earliest work in this field is thought to be the prediction of ionic sound in plasmas by Tonks and Langmuir in 1929.[1] In their paper, experimental observations are described which, however, do not clearly show the dispersion relation. The fundamental frequency and first and second overtones were observed in a mercury vapor discharge confined in a spherical discharge tube, by Revans in 1933.[9] Revans also showed evidence for radial standing ionic sound waves in a cylindrical tube. Longitudinal standing ionic sound waves were apparently observed in a magnetically supported plasma column by Alexeff and Neidigh.[10] Other standing-wave modes in a plasma column were seen by Consoli, LeGardeur, and Slama.[11] Data from the discharge experiments of Crawford[12] were analyzed

[9] R. W. Revans, Phys. Rev. **44**, 798 (1933).
[10] I. Alexeff and R. V. Neidigh, Bull. Am. Phys. Soc. **6**, 309 (1961).
[11] T. Consoli, R. LeGardeur, and L. Slama, Compt. Rend. **253**, 1923 (1961).
[12] F. W. Crawford, Stanford University Microwave Laboratory Report, ML-762, 1960 (unpublished).

by Alexeff and Neidigh[13] and by Moore[14] and were shown apparently to exhibit ionic sound waves. Additional work by Crawford,[15] with further corrections by Alexeff and Neidigh,[16] confirms this analysis. An investigation of standing ionic sound waves in spherical discharge tubes was made by Alexeff and Neidigh.[17] Standing ionic sound waves in a spherical plasmoid were observed by Geller and Lucarain.[18] The gas damping of standing ionic-sound-wave modes was used to estimate the gas pressure in the arc-type ion source of a magnetic isotope separator by Alexeff, Veach, and Yonts.[19] Traveling ionic sound waves were produced in discharge tubes by Little[20] and by Hatta and Sato.[21]

IV. OBSERVATION OF IONIC SOUND OSCILLATIONS IN A MAGNETICALLY SUPPORTED PLASMA COLUMN

A. Construction and Operation of the Apparatus

A schematic diagram of the apparatus which produced the plasma column is shown in Fig. 1. Primary electrons emitted from a negative, incandescent tantalum filament flowed to a grounded anode of carbon or of copper. Some primary electrons passed through a 1/4-in. aperture in the anode and were collimated by a magnetic field. Opposite the cathode was an insulated anticathode, which remained at cathode potential. Thus, the primary electrons in the collimated beam were reflected at the anticathode end and presumably made several oscillations from anticathode to cathode before reaching the anode.

The plasma-column ions and secondary electrons appeared when the background pressure in the system was raised to about 5×10^{-4} Torr. The plasma column had a maximum density of about 10^{13} ions/cm^3 for a total cathode emission current of several amperes. A typical cathode operating potential was -50 V. The average density was measured by turning off the incident electron stream, sweeping out the plasma along lines of magnetic flux, and collecting it with a "plasma sweeper."[22] The plasma density appeared to be uniform along the magnetic field. The magnetic fields used in the experiments ranged from 1000 to 7000 G.

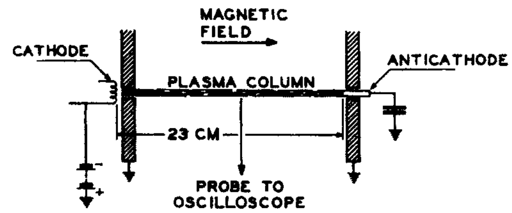

FIG. 1. Apparatus for producing the magnetically supported plasma column.

The plasma column was found to be oscillating. An electrostatic probe thrust into the plasma showed sinusoidal oscillations up to 20 V in amplitude. The oscillations also appeared in the light emitted by the plasma column when it was observed with a photomultiplier. Typical oscillations are shown in the multitrace oscillogram of Fig. 2, top. The frequency of the oscillation was grossly independent of the magnetic field, the filament potential, and the background gas pressure. The filament temperature had the most effect on the frequency, which could be altered or "pulled" slightly but which would change discontinuously to a higher frequency if altered too much. In this way, the frequency shown in Fig. 2, top, was made to change to the frequency shown in Fig. 2, bottom. In general, the apparatus was adjusted to produce the clearest sine wave of the largest possible amplitude.

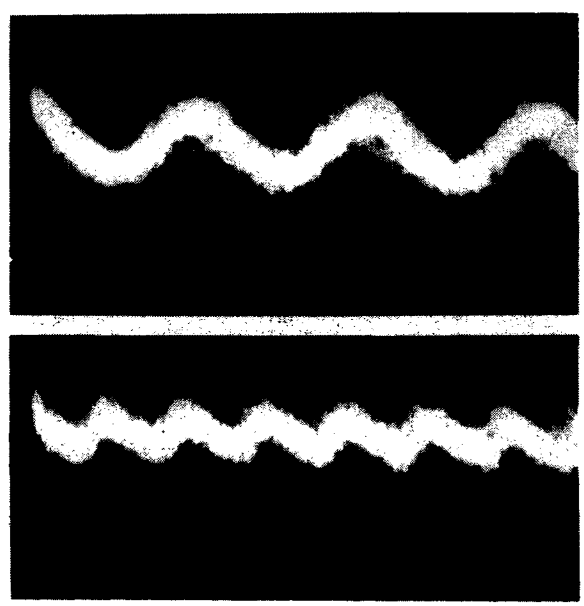

FIG. 2. Fundamental and first overtone in a xenon plasma column 23 cm long 20 μsec/cm, 0.2 V/cm.

[13] I. Alexeff and R. V. Neidigh, Oak Ridge National Laboratory Thermonuclear Project Semiannual Report, ORNL-3104, 1961 (unpublished), pp. 40–45.
[14] Acknowledged as private communication in reference 15.
[15] F. W. Crawford, Phys. Rev. Letters **6**, 663 (1961).
[16] I. Alexeff and R. V. Neidigh, Oak Ridge National Laboratory Thermonuclear Project Progress Report, ORNL-3239, 1961 (unpublished), pp. 28–29. (The correction corresponds to noting that the frequency which Crawford finds at roughly twice the fundamental frequency is not necessarily produced by nonlinear amplification of the fundamental. The frequency probably corresponds to the previously ignored second overtone of the system, which lies at 2.14 times the fundamental frequency.)
[17] I. Alexeff and R. V. Neidigh, Phys. Rev. Letters **6**, 223 (1961).
[18] M. Geller and M. Lucarain, Compt. Rend. **253**, 1542 (1961).
[19] I. Alexeff, A. M. Veach, and O. C. Yonts, Bull. Am. Phys. Soc. **7**, 442 (1962).
[20] P. F. Little, Nature **194**, 1137 (1962).
[21] Y. Hatta and N. Sato, *Ionization Phenomena in Gases*, edited by H. Maecker (North-Holland Publishing Company, Amsterdam, 1962), Vol. 1, p. 478.
[22] I. Alexeff and R. V. Neidigh, Proceedings of the Third Annual Symposium on the Engineering Aspects of Magnetohydrodynamics, March 28–30, 1962, Rochester, New York (to be published).

B. Model of the Oscillating Plasma Column

The observed radio-frequency oscillations are interpreted in the following manner: The fixed frequency is assumed to be determined by a standing ionic sound wave, as given by Eq. (3). The wave extends along the plasma column, parallel to the magnetic field. For the fundamental mode of oscillation, the wavelength λ is twice the length of the plasma column L (in centimeters).

The electron temperature T_e is assumed to be fixed roughly by the first excitation potential of the un-ionized gas filling the apparatus. Electrons having energy greater than the first excitation potential, rapidly lose their energy by excitation and ionization processes. Electrons having energy less than the first excitation potential lose their energy much more slowly. During experiments with a gas, T_e should be roughly a constant despite changes in the discharge conditions. For different common gases, T_e should not vary by more than a factor of 3.5, and the variation of T_e from gas to gas should be also roughly predictable.

The ion temperature in the plasma column was not measured in these experiments. It was assumed to be much lower than the electron temperature. In previous experiments with a similar column, the ion temperature was a few electron volts.[23]

The value of the constant γ can be 1, 5/3, or 3. In these plasma-column experiments, $\gamma=3$ empirically produces the best agreement between the measured values of ν and T_e. The value of γ is further discussed in the next section, Sec. IV C.

The above model of the oscillating plasma column predicts the following fundamental frequency:

$$\nu=(1/2L)(\gamma kT_e/m_i)^{1/2}. \qquad (12)$$

Here ν is the observed frequency in cycles per second, L is the length of the plasma column in centimeters, γ is 3, k is Boltzmann's constant in ergs per °K, T_e is the electron temperature in °K, and m_i is the ion mass in grams. The value of T_e roughly corresponds in energy to the lowest excitation potential in the un-ionized gas.

If the model of the oscillating plasma column is correct, then the frequency should behave as follows:

1. The frequency can be pure (sinusoidal waveform).
2. The frequency is not a function of the magnetic field or of the ion density. (Thus the frequency should be independent of the gas density in the apparatus, even though the plasma density varies with gas density.)
3. The frequency is independent of plasma supply voltage and plasma supply current.
4. Higher frequencies or overtones occur as integral multiples of the fundamental frequency.
5. The frequency is proportional to L^{-1}, to $m_i^{-1/2}$, and to $T_e^{1/2}$.

[23] J. R. McNally and M. R. Skidmore, J. Opt. Soc. Am. **47**, 863 (1957).

C. Experimental Confirmation of the Model

The predictions of the model of the oscillating plasma column appear to be valid. The first three predictions fit the preliminary observations: The frequency can be pure (see Fig. 2), the frequency is grossly independent of magnetic field and of gas density, and the frequency is grossly independent of plasma supply voltage and current.

Overtones were studied next. The oscillation would abruptly change in frequency if the parameters of the discharge were properly varied. The fundamental and the first overtone of a column of xenon plasma are shown in Fig. 2. The discharge conditions were almost identical in the two cases. The ratio of the frequencies is almost 1 to 2, as expected. On occasion, overtones up to the fourth were observed. Thus the prediction that overtones should appear as integral multiples of the fundamental frequency is verified.

The predicted dependence of frequency on the length of the plasma column appears to be correct. In some experiments, the length of the column was varied while the system was operating. The results for an argon plasma are shown in Fig. 3. As predicted, the frequency is proportional to L^{-1}.

The predicted dependence of frequency on ion mass also appears to be correct. The ion mass varied from mass 1 (hydrogen) to mass 201 (mercury). Typical results are shown by the lower set of points in Fig. 4. The line labeled "23-cm arc" is given by the ionic-sound-wave formula of Eq. (12) if $\gamma=3$, T_e is a constant = 17 eV, and L is the length of the column = 23 cm. As predicted, the frequency is proportional to $m_i^{-1/2}$.

More evidence supporting the predicted dependence of frequency on the length of the plasma column is obtained from the upper set of points in Fig. 4. In this case, a plasma column 10.5 times shorter than the previously discussed column was used. As predicted, all frequencies are 10.5 times higher. However, too close a comparison of the two plasma columns is probably not justified, because the long column terminated on

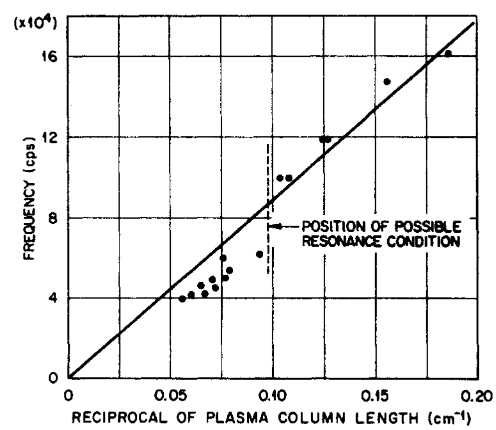

Fig. 3. Frequency dependence on plasma-column length. An argon plasma was used.

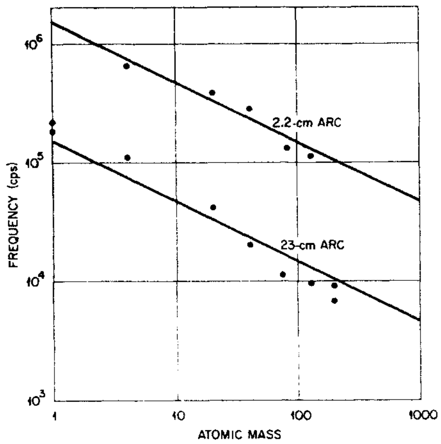

FIG. 4. Frequency dependence on ion mass for the plasma column.

an anticathode electrode, while the short column terminated on a free surface. The short, free-surface plasma column is locally referred to as a "pressure-gradient or mode II arc" and is further discussed in Sec. VII of this paper.

The predicted dependence of frequency on temperature has not been fully investigated. Two basic difficulties are present: First, the electron temperature in the discharge apparently does not vary much. Thus the data in Figs. 3 and 4 can be fitted by a theoretical line, assuming constant temperature. A second difficulty is that an independent measurement of the electron temperature is difficult. Spectroscopic measurements did not appear to give valid electron temperatures, probably because the electron energy distribution was not actually Maxwellian. Langmuir-probe temperature measurements have yielded complex curves that are difficult to interpret. Thus, observing the variation of the ionic-sound-wave frequency as a function of the electron temperature was not possible.

However, on the average, the electron temperature obtained by using the ionic-sound-wave model appears to be correct.[24] The ionic-sound temperature measurements agree within a factor of 2 with the Langmuir-probe temperature measurements. Also the ionic-sound temperatures correspond well with the lowest excitation potentials in the residual gas. For example, the theoretical lines in Fig. 4 correspond to an electron temperature of 17 eV. The lower set of experimental points corresponds to an average excitation potential of 12 eV; the upper set, 14 eV. The gases used were H_2, He, Ne, Ar, Kr, Xe, and Hg.

The numerical value of the adiabatic compression coefficient γ presents a difficulty. The value $\gamma=3$ yields electron temperatures from the ionic-sound frequency measurements which best agree with the temperatures found by other techniques. However, the value $\gamma=5/3$ is more consistent with the high rate of collision between the electrons and both the ions and the background gas atoms. Thus in argon at 5×10^{-4} Torr, a 17-eV electron collides about 3×10^6 times per second with neutral argon atoms. This frequency of collision is about three times higher than the highest observed ionic-sound frequency. Thus $\gamma=5/3$ is always expected. Why the apparently incorrect value of $\gamma=3$ yields the best value for the electron temperature is not known.

[24] I. Alexeff and R. V. Neidigh, Oak Ridge National Laboratory Thermonuclear Project Progress Report, ORNL-3239, 1961 (unpublished), pp. 27–28.

D. Direct Evidence for Plasma Pulsations Along the Magnetic Field

Evidence which suggests that the plasma was actually pulsating along the lines of magnetic flux was obtained with the pressure-gradient or mode II plasma column. (See Sec. VII for a description of this apparatus.) The plasma column extended away from the cathode into the vacuum chamber, where it visually appeared to terminate in a free surface. The light output at the free surface was observed with a photomultiplier and an associated optical system.

Evidence suggesting that the plasma column lengthens and shortens symmetrically is shown in the upper oscillograms of Fig. 5. Light signals from the

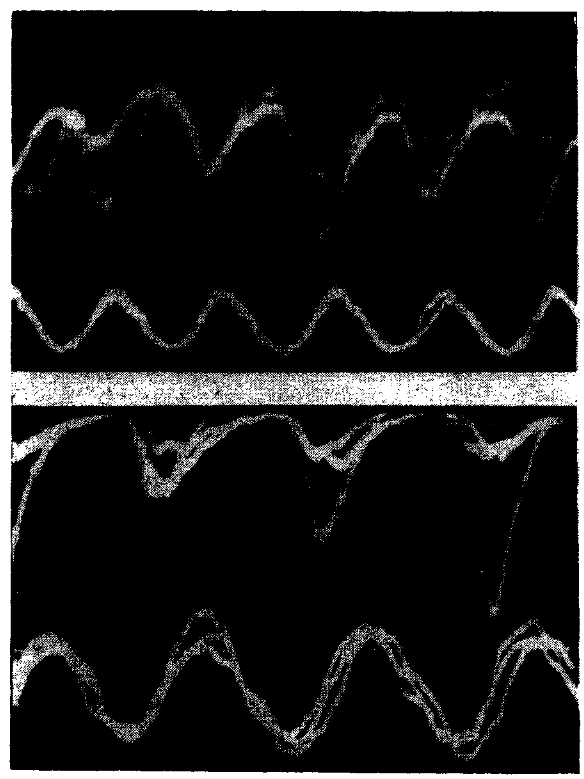

FIG. 5. Pulsations in a pressure gradient plasma column. Upper traces on the upper oscillogram correspond to light pulses from the cathode and the anticathode end of the plasma column. Upper traces on the lower oscillogram correspond to light pulses from the top, center, and bottom of the anticathode end of the plasma column. All pulses are in phase. The lower pulses in both oscillograms are reference signals from an electrostatic probe.

cathode and the anticathode ends of the plasma column are shown by the two upper single traces. Zero light intensity corresponds to the topmost line. The light pulses downward are in phase, suggesting that the plasma is pulsing in and out. The lower traces in this oscillogram are reference signals from an electrostatic probe which triggered the oscilloscope.

More evidence suggesting that the plasma column is pulsing is given in the lower oscillogram of Fig. 5. The upper three traces represent light from the top, center, and bottom of the anticathode end of the plasma column. Again, the light pulses are in phase, suggesting that the end of the plasma column is pulsing. The lower traces are again reference signals from a probe.

Although the above two pieces of information suggest that the plasma is pulsing, another explanation is possible. The plasma might not be moving as much as changing its light output due to variations in the cathode current. However, the above information does definitely show that some types of plasma motion did not occur in this particular plasma column. Rotation, for example, is excluded.

E. Effect of Gas Damping

The effect of gas damping on the plasma-column oscillations is evaluated to make sure that the oscillations are permitted. The Q of the plasma oscillation is given by Eq. (11):

$$Q=4\pi\nu/\nu_c.$$

For a typical case (argon in the 23-cm-long plasma column), the observed fundamental frequency of oscillation ν is 2×10^4 cps. The rate of collision of an argon ion with argon atoms ν_c is obtained from Eq. (7):

$$\nu_c=\sqrt{2}\sigma n_g(1.13)(2kT/m)^{1/2}.$$

The argon ion-argon atom scattering cross section σ is obtained from the argon atomic radius[25] of 1.54×10^{-8} cm, assuming that the ion has the same radius. The total scattering cross section is $\sigma=4\pi r^2$, where r is the ionic and the atomic radius in centimeters. The value of σ is computed to be 3.0×10^{-15} cm^2. The number of argon atoms per cubic centimeter n_g is computed from the operating pressure of 5×10^{-4} Torr.

Since the ion temperature was not measured, upper and lower limiting values are used to compute upper and lower limits for Q. An upper limit for Q is 78 if the ions are at the same temperature as the gas atoms, 1/40 eV. A lower limit for Q is 4.3 if the ions have the same temperature as the electrons, 17 eV. As the true value of Q lies between the two limiting values $4.3<Q<78$, ionic sound oscillations are permitted.

A practical application of gas damping was in measuring the gas pressure in an arc-type ion source of a magnetic isotope separator.[19] The damping away of a given standing-ionic-sound-wave mode was used as a crude estimate of the gas pressure in the ion source. Such a pressure estimate is useful when a direct measurement is difficult because the ion source is operating with a vaporized solid.

F. Discussion of Experiments with Plasma Columns

The ionic-sound-wave model of the oscillating, magnetically supported plasma column appears to fit all the experimental data so far obtained. Sinusoidal oscillations are obtained which (1) show the predicted overtones, (2) have the correct frequency dependence on plasma-column length and on ion mass, and (3) correspond to the correct electron temperature. Direct evidence suggests that the plasma column is oscillating along magnetic field lines. The computed effect of gas damping on the oscillations of the plasma is not serious.

V. OBSERVATION OF IONIC SOUND OSCILLATIONS IN SPHERICAL DISCHARGE TUBES

A. Construction and Operation of Apparatus

As a check of the existence of ionic sound waves, they were studied in a second system, one which had no externally imposed magnetic field. The system chosen was a spherical glass discharge tube instead of the usual cylindrical tube. It was thought that the lowest mode of oscillation in a sphere might be less damped, as this mode involves no transport of matter parallel to the wall.

The construction of the spherical discharge tube is shown in Fig. 6. The glass sphere had a hot cathode of tungsten wire and a metal disk as an anode. Both electrodes were flush with the wall, and on opposite ends of a diameter. The electron temperature in the plasma was measured by a Langmuir probe in the center of the sphere. A gas pressure of about 10^{-2} Torr was used. Discharge voltages ranged from about 20 V to over 100 V and discharge currents varied from about 1 mA to 5 A.

Oscillations of about 1-V amplitude were found on a capacitively coupled electrode outside the sphere. Varying the discharge conditions caused the oscillation frequency to change discontinuously from mode to mode in a most striking manner. Oscillograms of an apparent

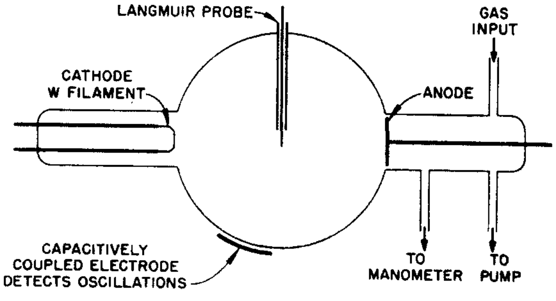

FIG. 6. Construction of the spherical discharge tubes.

[25] *Handbook of Chemistry and Physics* (Chemical Rubber Publishing Company, Cleveland, Ohio, 1952), 34th ed. p. 2877.

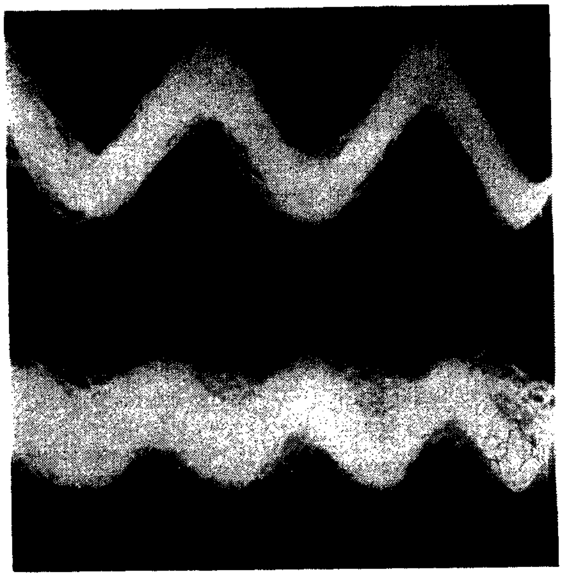

FIG. 7. Apparent fundamental frequency and first overtone in a helium plasma 1.0 μsec/square division, 0.5 V/square division. A spherical discharge tube 10 cm in diameter was used.

fundamental frequency and a first overtone are shown in Fig. 7. The lower frequency modes of oscillation were extremely difficult to excite. Probably a slight asymmetry in the sphere preferentially excited higher, asymmetric modes of oscillation, and the high gas pressure preferentially damped the lower ones.

B. Model of the Oscillating Sphere

The fundamental frequency in an oscillation sphere of plasma, obtained by applying Eq. (3), is

$$\nu=(1/CD)(\gamma kT_e/m_i)^{1/2}. \tag{13}$$

In this formula, ν is the observed frequency in cycles per second, D is the sphere diameter in centimeters, γ is the adiabatic compression coefficient, k is Boltzmann's constant in ergs/°K, T_e is the electron temperature in °K, and m_i is the ion mass in grams. The ion temperature was assumed to be small compared with the electron temperature. This assumption is probably a good one for a discharge tube operating at a relatively high pressure, because collisions between ions and gas atoms are frequent. The factor C is a dimensionless constant, which is 1.00 if the plasma boundary is a pressure node and 1.51 if the boundary is a pressure antinode. If the first-mentioned boundary condition is valid, the first and second overtones have frequencies 1.43 and 1.84 times the fundamental, respectively. If the second boundary condition applies, the ratios are 1.61 and 2.16, respectively.[26]

[26] J. W. S. Rayleigh, *The Theory of Sound* (Dover Publications, New York, 1956), Secs. 331 and 332.

C. Experimental Verification of the Model

The dependence of the lowest observed frequency on ion mass m_i and on wavelength λ (sphere diameter D) was observed, as was done with the oscillating plasma column. The ion mass was varied from mass 1 (hydrogen) to mass 131 (xenon). Spheres of 2.5-, 10-, and 25-cm diameter were used.

The dependence of the lowest observed frequency on ion mass m_i for a sphere 10 cm in diameter is shown in Fig. 8.

All observed frequencies were normalized to an electron temperature of 1 eV, so that the frequency dependence on mass would be clearly visible. During each run, the electron temperature T_e was measured with the Langmuir probe. The observed frequency was then normalized by the following formula: $\nu_{\text{normalized}}=(T_e)^{-1/2}\nu_{\text{observed}}$. The values of $\nu_{\text{normalized}}$ should be independent of electron-temperature variations from gas to gas and from run to run. Although the experimental points of Fig. 8 show much scatter, they do indicate that the frequency is proportional to $m_i^{-1/2}$.

The dependence of lowest observed frequency on sphere diameter D is shown in Fig. 9. Each of the seven points for a given sphere diameter corresponds to a different ion mass, the ion masses being those shown in Fig. 8. The observed frequency for each ion mass is multiplied by $m_i^{1/2}T_e^{-1/2}$, so that only the frequency dependence on sphere diameter should be present. Although the points show much scatter, the frequency does appear to be proportional to D^{-1}.

The value of the electron temperature T_e can be found from the fundamental frequency by use of Eq. (13). However, three difficulties are present: First, the value of the boundary-condition constant C (1.00 or 1.51) is not known. Second, the value of the adiabatic compression coefficient (1, 5/3, or 3) is not known. Third, the lowest observed frequency is probably not always the fundamental frequency, as the lowest frequencies are very difficult to excite. Therefore, all the

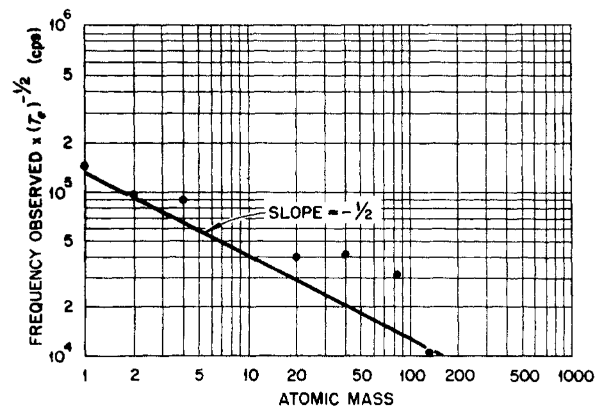

FIG. 8. Frequency dependence on ion mass. A spherical discharge tube 10 cm in diameter was used.

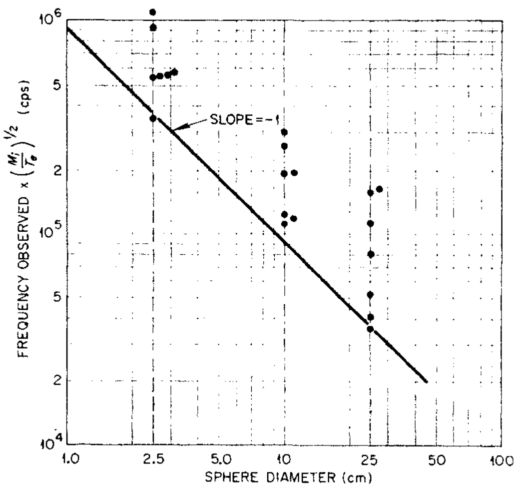

FIG. 9. Frequency dependence on sphere diameter.

experimental points (a total of 21) were studied statistically to obtain the value of C and of γ.

The experimentally observed frequencies were reduced so that runs with different gases, different sphere diameters, and different electron temperatures could be compared directly. The formula for a reduced frequency is

$$\nu_{\text{reduced}} = \nu_{\text{observed}} (m_i/T_e)^{1/2} (D/10). \tag{14}$$

Here m_i is the atomic mass in amu, T_e is the measured electron temperature in eV, and D is the sphere diameter in centimeters. Thus the reduced frequency corresponds to a hydrogen plasma in a sphere 10 cm in diameter at a temperature of 1 eV. If in all 21 cases the sphere were oscillating at the fundamental frequency, and if γ and C remained constant, then all the values of ν_{reduced} would be equal.

The values of ν_{reduced} are plotted in Fig. 10. A considerable spread in the frequencies shows that possibly overtones are observed and that possibly γ or C varied. For comparison, the theoretical fundamental frequency is also plotted for $\gamma = 1$, 5/3, and 3, and for $C = 1.00$ and 1.51.

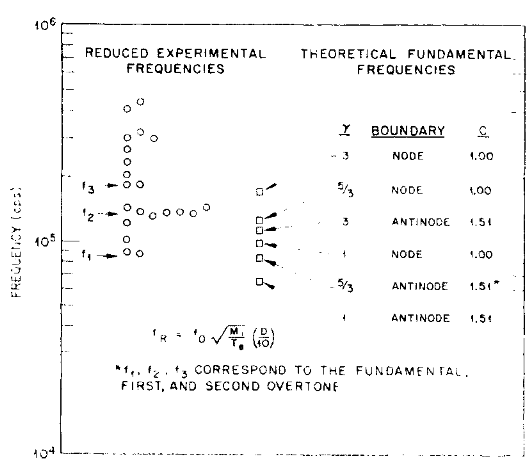

FIG. 10. Reduced experimental frequencies and theoretical fundamental frequencies for the spherical discharge tubes.

The theoretical fundamental frequencies roughly correspond to the lower reduced experimental frequencies. This correspondence is evidence that the spherical discharge tubes were oscillating in the lower ionic sound modes.

Of the 21 values of ν_{reduced}, seven appear to form a well-defined group as shown in Fig. 10. This group can either correspond to the theoretical fundamental frequency for $\gamma = 5/3$ and $C = 1$, or to the first overtone for $\gamma = 5/3$ and $C = 1.51$. In the latter case, the theoretical fundamental frequency corresponds closely to the few lowest values of ν_{reduced}. The theoretical fundamental frequencies having $\gamma = 3$ are improbable because of the high electron-gas atom collision rate. The values of $\gamma = 5/3$ and $C = 1.51$ are suspected to be correct, but a definite conclusion requires more experimental data.

An attempt to classify the overtones which were observed is shown in Table I. In this table are listed the frequency ratios which were observed in a given run, when the discharge tube changed its mode of oscillation. The observed frequency ratios appear to agree with those predicted if the constant C is 1.51, although classification was difficult because the lowest modes of oscillation were not always excited.

D. Effects of Gas Damping

Gas damping of the lower frequency modes of oscillation in the spherical discharge tubes appears to be serious. Gas damping is probably one reason for the infrequency of observation of the fundamental mode.

As an example, the damping of the fundamental mode in an argon-filled discharge tube 10 cm in diameter is evaluated. Typical operating parameters are a gas pressure of 10^{-2} Torr, an electron temperature of 3 eV, and a fundamental frequency of about 3.3×10^4 cps. The quality factor Q of the oscillating system is computed from Eq. (11):

$$Q = (4\pi\nu/\nu_c).$$

TABLE I. Ratios of frequencies observed in the spherical discharge tubes. The calculated ratios are given in the left column. The lowest frequency observed for helium, neon, and argon was thought to be the first overtone because this assumption provided a better fit to the data

$C = 1.51$	H	He	Ne	Ar	Kr	Xe
1.00	1.00				1.00	1.00
1.61	1.46	1.61	1.61	1.61	1.75	1.57
2.16			1.93		2.00	2.16
2.17		2.30	2.22	2.20		
2.71		2.72			2.59	2.63
2.85						
3.25			3.43			

For an upper limit for Q, the ion temperature is assumed to equal room temperature 1/40 eV. The temperature of the argon atoms is also assumed to be 1/40 eV. The argon ion-argon atom collision cross section is assumed to be the same as that used in Sec. IV E: 3.0×10^{-15} cm^2. Under these assumptions, the rate of collision between an ion and gas atoms is 8.4×10^4 collisions per second, and the Q of the fundamental mode of oscillation is 5.

The upper limit of 5 computed above for Q is sufficiently close to the limiting value of 1/2 to make one expect damping of the fundamental mode of oscillation in the argon-filled sphere. The gas damping is even more severe if the ion temperature is higher than the assumed value of 1/40 eV. Increasing the value of Q by decreasing the background gas density was not possible, as at lower pressures the glow discharge was not maintained.

The effect of gas damping on the oscillating discharge was studied by increasing the gas pressure. As the pressure was increased, the lowest possible modes of oscillation were progressively damped away. This observation is consistent with Eq. (11), which states that the higher the frequency of a given mode, the higher the Q of that mode, and that any given mode can be damped away if the ion-atom collision rate is increased sufficiently.

E. Observation of Ordinary Sound Waves

When the gas pressure in the discharge tube was raised to about 1 Torr, an unexpected result occurred. Oscillations suddenly appeared which were about 10 times lower in frequency than that of the fundamental plasma mode. These oscillations correspond in frequency to ordinary standing sound waves in the background gas. At 1 Torr, the background gas is dense enough to support ordinary sound waves having the required wavelength of several centimeters, because the mean free path for atom-atom collisions is about 0.01 cm. Apparently, ordinary gas oscillations carry the plasma with them and thus produce detectable electrical oscillations.

At the high pressure of 1 Torr, plasma oscillations for high-frequency modes can still occur, superimposed on the ordinary sound wave oscillations. Thus a very confusing noise spectrum may be observed in a discharge tube at excessively high pressure.

F. Discussion of Experiments with Discharge Tubes

The ionic-sound-wave hypothesis appears to fit all the experimental data so far obtained with the spherical discharge tubes. Sinusoidal oscillations are predicted and are obtained. These oscillations show overtones and the correct dependence on sphere diameter and on ion mass, among other predicted features. The fundamental mode of oscillation yields a frequency corresponding roughly to that predicted from the measured electron temperature. Extra effects appear because of the high background-gas pressure needed to maintain the discharge in the tube. However, these extra effects are easily identified as gas damping of the plasma oscillations and as ordinary sound oscillations in the background gas.

VI. MECHANICAL MEASUREMENT OF PLASMA-ELECTRON PRESSURE

The restoring force producing the ionic sound wave is derived from the thermal pressure of the hot plasma electrons, as is discussed in Sec. II B. The existence of the thermal electron pressure was verified by measuring it directly by means of a simple mechanical method. The method of measurement is only briefly outlined here, as the experiment is discussed in detail elsewhere.[27]

The apparatus used is shown schematically in Fig. 11 The plasma used was contained in the plasma column described in Sec. IV. At its anticathode end, the force exerted by the plasma column was measured by a sensitive torsion balance. The torsion balance was electrically insulated so that no current flow could disturb the measurements.

The basic assumption in this work is that the force exerted by the end of the plasma column is due only to the plasma electrons. In this case, the force on the balance is given by

$$F=An_ekT_e. \tag{15}$$

Here the force F is in dynes, A is the cross section of the plasma column in square centimeters, n_e is the average number of electrons per cubic centimeter, k is Boltzmann's constant in ergs per °K, and T_e is the electron temperature in °K.

To verify that the experimentally measured plasma-electron pressure was actually being observed, the observed force was compared with a computed value. The computed value is found from Eq. (15), if one knows the plasma-column cross section A, the plasma-electron density n_e, and the plasma-electron temperature T_e. In this experiment, the plasma-column cross

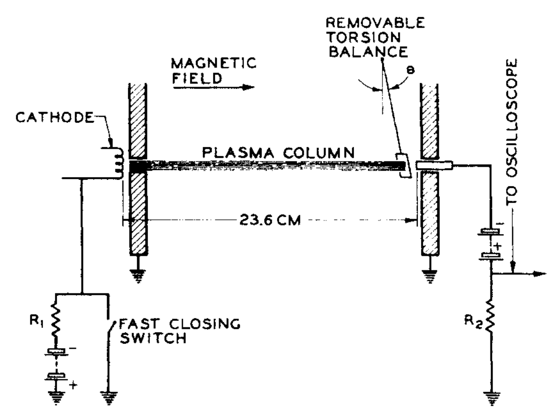

Fig. 11. Apparatus for the measurement of plasma-electron pressure.

[27] I. Alexeff and R. V. Neidigh, Phys. Rev. 127, 1 (1962).

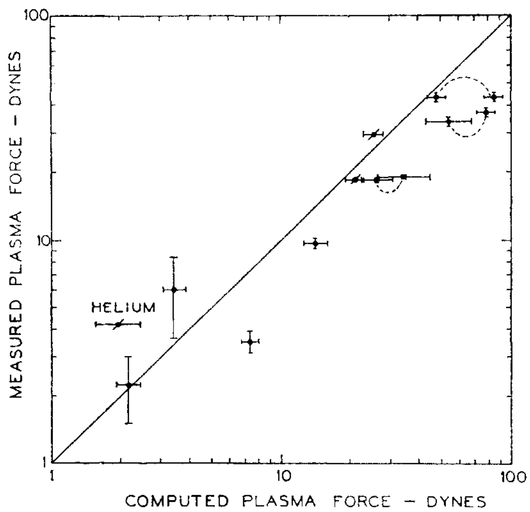

FIG. 12. Experimental plasma force vs computed plasma force. All unlabeled points are for argon.

section A was assumed to be the cross section of the visible column. The electron density was measured with a "plasma sweeper."[22] In using the plasma sweeper, the source of plasma is turned off and the plasma filling the column is "swept out," collected, and measured. The average plasma density is computed by dividing the amount of plasma in the column by the volume of the plasma column. The electron temperature was obtained both by Langmuir-probe measurements and by using ionic sound waves. Ionic sound waves may be used to measure electron temperature because the apparatus is a self-excited ionic-sound-wave oscillator, as described in Sec. IV. By measuring the fundamental frequency of oscillation of the system, and by applying Eq. (12), one finds the electron temperature. Experiments indicate that the ionic-sound-wave method of measuring temperature gives satisfactory results.[24] For a detailed discussion of the measurements, the reader is referred to the original paper.[27]

The results of the experiment are shown in Fig. 12. The vertical error bars represent the drift between a force measurement at the beginning of a run and one at the end. A diagonal line means that only one force measurement was made. The horizontal error bars correspond to the known errors in the computed force.

The results indicate that the measured force on the torsion balance agrees roughly with the computed force obtained from the experimentally measured electron density and electron temperature of the plasma. The large range of agreement between the measured force and the computed force, 2 to 40 dyn, implies that in this apparatus, at least, the force exerted on the balance is the force of the thermal electron pressure, which can be measured directly.

VII. IONIC SOUND RESONANCE WITH THE ION CYCLOTRON FREQUENCY

Resonance effects have been observed between the longitudinal ionic sound waves and the transverse ion cyclotron rotation in the oscillating plasma column described in Sec. IV. No detailed study of the resonance effects has been made. However, the observations of the effects are briefly described below.

An interesting resonance effect is shown in Fig. 3, Sec. IV. These data show how the frequency of the oscillating plasma column varies as a function of the column length. A discontinuity in the experimental points appears to be present where the ionic-sound-wave frequency equals the ion cyclotron frequency. One also finds that the floating potential of a probe in the secondary plasma increases when the ionic-sound-wave frequency drops. The cause of this frequency change is not known at present.

Another interesting resonance effect is the forced transition between a long plasma column and the short pressure-gradient ("mode II") plasma column. The ordinary plasma column (mode I), occurs when gas is fed into the entire vacuum tank. The short pressure-gradient column occurs when gas is fed into the electron-beam-defining aperture adjacent to the filament.[28] A high vacuum is maintained in the rest of the system. Under these conditions, a localized volume of plasma is formed in the beam-defining aperture alone. This localized plasma volume forms a short column which appears to terminate in a free surface. By feeding gas at varying rates into both the electron-beam-defining aperture and the entire vacuum tank, one can induce a transition from the long to the short plasma column and inversely.

With proper adjustment of the gas feed, cathode voltage, and cathode current, the plasma column became unstable. The column would change abruptly from the long column to the short column as the magnetic field was reduced. As the magnetic field was increased, the reverse change occurred. The change occurred for a magnetic field at which the ion cyclotron frequency equalled the ionic-sound-wave frequency in the short, mode II plasma column.

Some experimental measurements of the long-plasma-column–short-plasma-column transition are tabulated below. The ionic-sound-wave frequency was measured by an electrostatic probe near the cathode.

Gas	Ion mass	Magnetic field for transition (G)	Cyclotron frequency at transition (kc/sec)	Ionic-sound frequency (kc/sec)
A	40	3750	147	160
N	28	3000	168	181
He	4	1100	430	520

[28] R. V. Neidigh and C. H. Weaver, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), Vol. 31, p. 315.

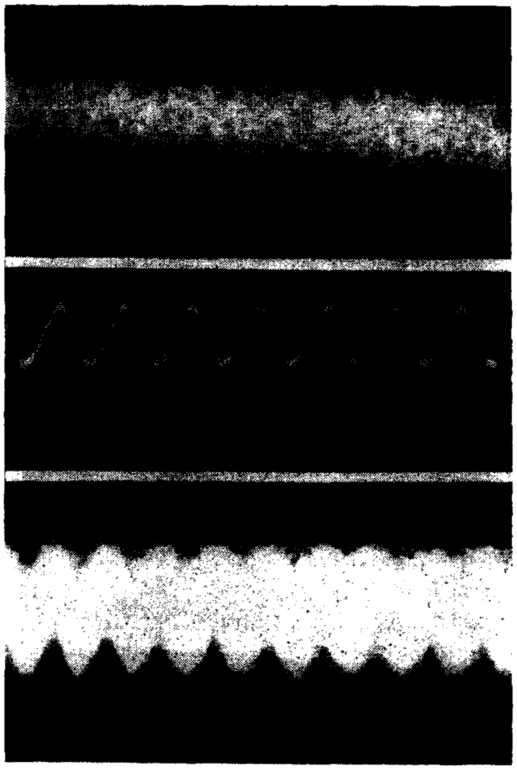

FIG. 13. Electrostatic probe signal during the long-plasma-column–short-plasma-column transition. Magnetic field: top—5000G, center—3750 G, bottom—1750 G. Argon gas was used. 5 μsec/cm, 2 V/cm.

The appearance of the signal on the electrostatic probe is shown in Fig. 13. The upper oscilloscope trace is for the long plasma column, which extends from the cathode to the electrically floating anticathode. The probe shows mostly random noise. At 3750 G the signal becomes remarkably simple harmonic (middle trace). Visually, the plasma column shortens from its initial long length of 23.0 cm to the short, pressure-gradient length of 2.2 cm. Further reduction in the magnetic field destroys some of the harmony in the signal (lower trace), but the short, pressure-gradient form of the plasma column persists.

The transition from the long to the short plasma column induced by a change in the magnetic field could be an ionic sound-ion cyclotron frequency resonance effect. However, the transition has also been interpreted in terms of the theory of plasma-column instability of Kadomtsev and Nedospasov[29] by Guest and Simon.[30]

In general, the resonance effects that have been observed have been small or have appeared in unstable systems. Usually the ionic sound wave oscillations are the dominating effect and are therefore quite easily studied.

VIII. CONCLUSIONS

A large number of observations have been made which indicate that ionic sound waves are easily produced and observed. Self-excited ionic-sound-wave oscillations were observed both in magnetically supported plasma columns and in spherical discharge tubes. The oscillations exhibit the expected overtones and the correct dependence on wavelength, ion mass, and electron temperature.

Further studies of the observed ionic sound waves reveal additional interesting effects. The effects of gas damping of the ionic-sound-wave oscillations were computed and experimentally verified. The source of the restoring force of the ionic sound waves is plasma-electron pressure of thermal origin. This pressure was measured mechanically. Finally, resonance effects between the ionic-sound-wave frequency and the ion cyclotron frequency in a magnetically supported plasma column were observed.

As a by-product of this work, several unusual methods of measuring the properties of a plasma have been tested. As discussed in Sec. VI, the electron temperature of the oscillating plasma column can be computed from the ionic-sound-wave frequency.[24] Use of the "plasma sweeper" yields the average electron density in the column.[22] Finally, measurement of the plasma-electron pressure yields the product of the electron temperature times the electron density.[27]

The effect of gas damping of the ionic sound oscillations can be quite serious. If the gas density in the magnetically supported plasma column were increased a hundred fold, damping of the fundamental mode of oscillation would be severe. In the spherical discharge tubes, the gas pressure required to maintain a discharge is so high that gas damping often prevents the fundamental mode of oscillation from appearing.

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[29] B. B. Kadomtsev and A. V. Nedospasov, J. Nucl. Energy, Part C, 1, 230 (1960).

[30] G. Guest and A. Simon, Phys. Fluids 5, 503 (1962).

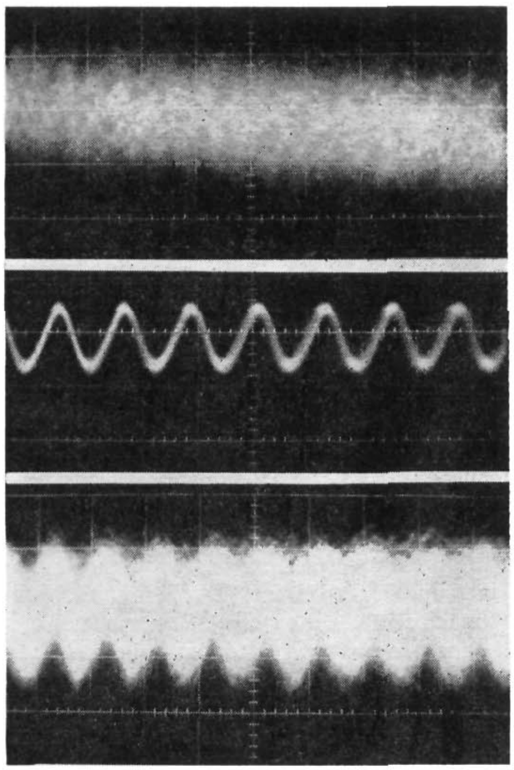

FIG. 13. Electrostatic probe signal during the long-plasma-column–short-plasma-column transition. Magnetic field: top—5000G, center—3750 G, bottom—1750 G. Argon gas was used. 5 μsec/cm, 2 V/cm.

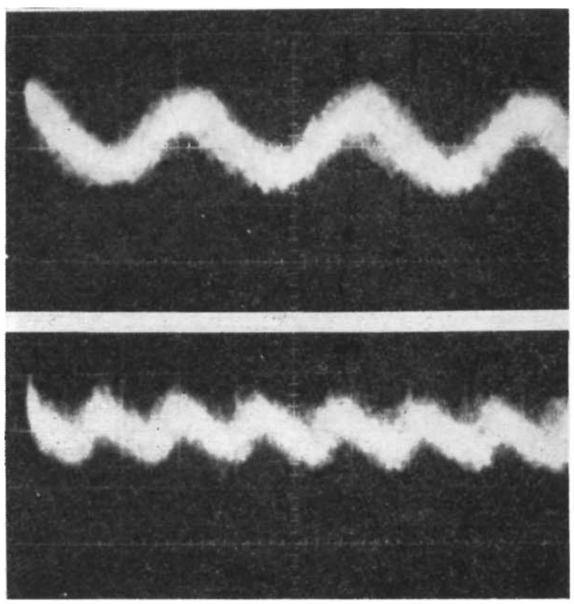

FIG. 2. Fundamental and first overtone in a xenon plasma column 23 cm long 20 μsec/cm, 0.2 V/cm.

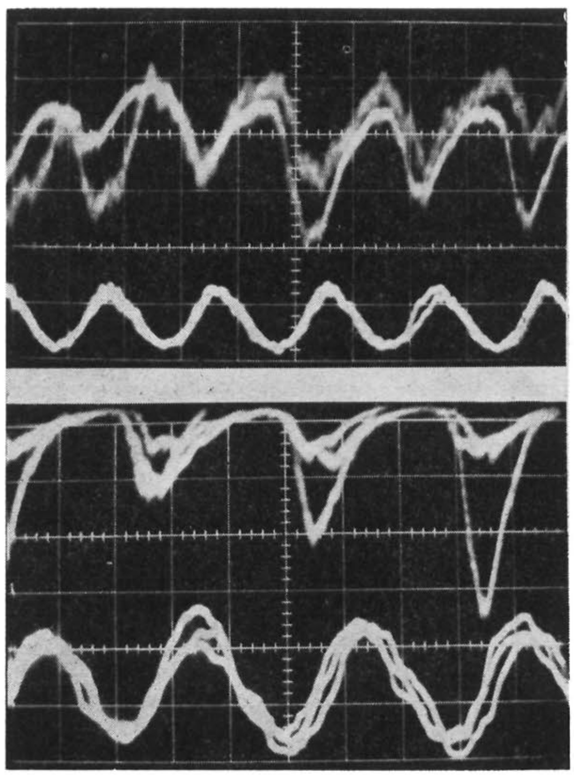

FIG. 5. Pulsations in a pressure gradient plasma column. Upper traces on the upper oscillogram correspond to light pulses from the cathode and the anticathode end of the plasma column. Upper traces on the lower oscillogram correspond to light pulses from the top, center, and bottom of the anticathode end of the plasma column. All pulses are in phase. The lower pulses in both oscillograms are reference signals from an electrostatic probe.

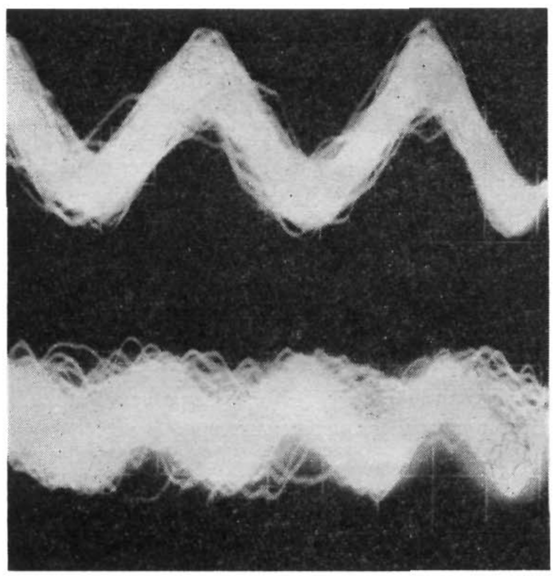

FIG. 7. Apparent fundamental frequency and first overtone in a helium plasma 1.0 μsec/square division, 0.5 V/square division. A spherical discharge tube 10 cm in diameter was used.